[CONTRIBUTION FROM THE BUREAU OF STANDARDS, U. S. DEPARTMENT OF COMMERCE.]

SOME CHARACTERISTICS OF THE GOUY THERMOREGULATOR.

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Received October 2, 1919.

Gouy[1] has described briefly a modification of the usual form of thermoregulator which produced a considerable improvement in the temperature regulation of a 100-liter water bath. Barnes[2] also made favorable comment on this device as applied to his work on the heat capacity of water.

While seeking means of overcoming some of the difficulties encountered in the use of the thermo-regulator for precise work, the author in 1918 arrived at the use of an oscillating contact as one solution. Later, reference to Gouy's description of this device was found, but since no direct comparison was there drawn between the fixed and oscillating contact type of thermoregulator and no mathematical expression defining the operation of the Gouy modification had been published and since, furthermore, the device appears not to have received the attention and use which its excellence and simplicity warrants, it was thought that this discussion would be of general interest.

The usual type of thermoregulator used in laboratories for precise regulation of temperatures consists of a bulb filled with an expansive liquid and connected to a glass U-tube which is partially filled with mercury. The variation in position of the mercury meniscus is caused to vary the energy input to the bath by throttling the supply of gas to a burner, by changing the electrical energy supply to a heating coil, etc.

Fig. 1 shows the U-tube and electrical connections arranged for electrical control of temperatures. In the usual form of regulator the platinum wire A is fixed in position. A rise in temperature of the regulator bulb causes the meniscus B to make contact with A whereupon current flows through the relay circuits opening a contact at D, thus introducing an additional resistance R_1 in the heating circuit, or opening this circuit entirely.

Gouy's modification consists in imparting to the platinum wire A an oscillating motion of 20 seconds period along the axis of the tube. His arrangement of electrical circuits was such that the entire heating current was interrupted when separations between the points A — B occurred. In this manner energy was periodically supplied to the bath during that part of the cycle of motion of the platinum wire during which A and B were not in contact. An increase in the temperature of the thermoregulator bulb results in a rising of the meniscus B which causes a decrease in the length of time during which energy is periodically supplied

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[1] *J. Physique*, 6, 479 (1897).

[2] *Phil. Trans. Roy. Soc.*, 199, 208 (1902).

to the bath, thus tending to stabilize the bath temperature. Gouy reports that the use of this device greatly reduced the errors due to the compressibility of the thermostatic fluid and to the distortion of the mercury meniscus as it moved up or down. He also states that regulation to within 0.0002° as measured by an extremely sensitive alcohol thermometer was maintained over periods of some hours and that the oscillation of mean bath temperature produced by the variations in heating was imperceptible.

Fig. 1 shows the oscillating device used by the author of this paper. A platinum wire A fitted to a guide plug E which fits the capillary tube loosely is given a periodic oscillating motion along the axis of the capillary tube by means of the cam and spring mechanism shown. The range of this motion may be conveniently 0.1 cm. and the period of the motion one second. This period may be varied over a wide range without affecting the regulation perceptibly; the lower limit being set by the disturbance of the mercury surface at extremely short periods and the upper limit by the time lag of the thermoregulator with respect to the heater. This time lag may be defined as the number of seconds which would elapse

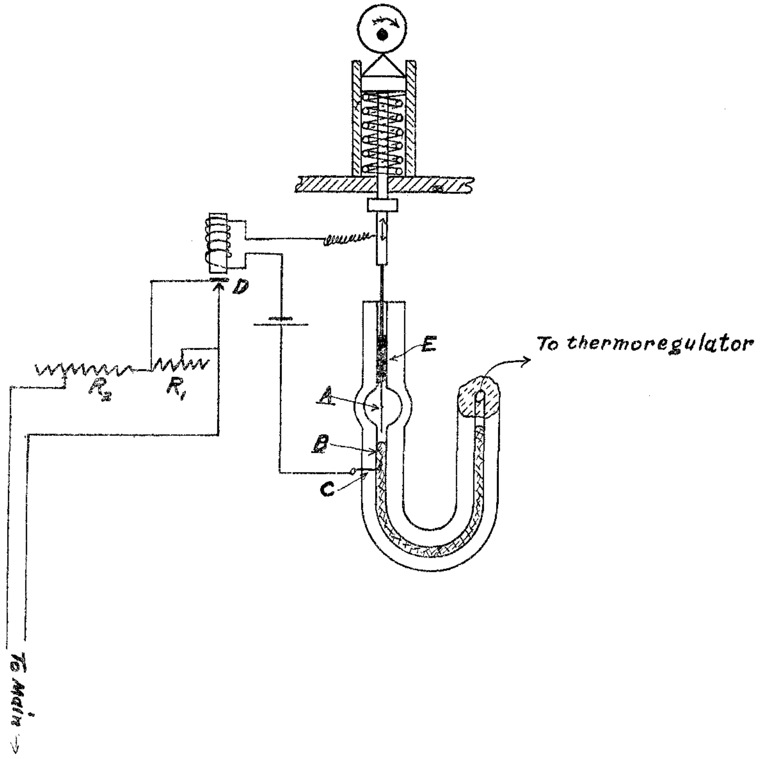

Fig. 1.

between the time when a certain quantity of heat, q, has been generated in the heater; the rate of heating being constant and continuous; and the time when the mercury meniscus of the thermoregulator has assumed a position corresponding to this quantity of heat.

The extent to which periodic variations of bath temperature due to operation of the regulator are reduced may best be illustrated by an experimental comparison between the usual or fixed contact type of thermoregulator and the Gouy modification or oscillating contact type, since this is not a matter which lends itself readily to analytical treatment.

The thermoregulator as illustrated in Fig. 1 was used as a fixed contact type to regulate the temperature of a water bath. By means of a very close adjustment of the external resistances, it was found possible to obtain a regulation where the up and down variation of bath temperature produced by the "on" and "off" action of the regulator was about 0.001–0.002 of a degree centigrade. This adjustment was not, however, practical since very slight changes in external conditions would cause the regulator to lose control. A practical adjustment, *i. e.*, one which would maintain control over a period of several hours under average laboratory conditions, was found to give a periodic variation of about 0.005°. The platinum wire was then given a reciprocating motion along the axis of the capillary with a periodicity of one second and it was found that the variation of bath temperature with operation of the regulator as indicated by a calorimetric platinum resistance thermometer was less than 0.0001°, even when the adjustment of the external resistances was such as would give a variation of approximately 0.01° with fixed contact. The above mentioned resistance thermometer was used with a galvanometer having a period of about 5 seconds. It is, therefore, probable that short period fluctuations of temperature considerably larger than the amount indicated (0.0001°) actually occurred. However, temperature fluctuations which are of too small period and amplitude to affect a calorimetric resistance thermometer would probably have a negligible effect in any but the most extreme cases. The periodicity of the motion was varied between 0.3 second and 3 seconds without observable effect upon the regulation. The time lag of the bath used was about 10 seconds. It is not to be understood that the operation of the regulation with fixed contact was the best obtainable with this type of regulator. In fact the mercury surface was known to be in bad condition, however, since the change to the oscillating contact was the only variation introduced, the comparison may be considered typical.

An analytical consideration of the operation of the two types of regulators (see appendix) results in the following characteristic equations. Symbols have the meanings indicated in appendix.

For fixed contact thermoregulator, the periodic change in temperature as the regulator operates is

$$\Delta\theta_\rho = \frac{tW}{M} + \Delta\theta'. \qquad (2)^{1}$$

The changes in mean bath temperature, $\Delta\theta_m$, produced by changes in thermal head, $\Delta\phi$, and by changes in maximum electrical input controlled by regulator, ΔW, is

$$\Delta\theta_m = t/2M.\Delta W + -tK\Delta\phi. \qquad (1)$$

The corresponding formulas for the oscillating contact regulator are:

For changes in mean bath temperature, $\Delta\theta_m$, due to changes in thermal head, $\Delta\phi$, and changes in average electrical distribution along the path of the moving contact ΔW,

$$\Delta\theta_m = \frac{MK\phi_1}{\alpha W_1 W_2}\Delta W - \frac{MK\Delta\phi}{\alpha W_2}. \qquad (5)$$

The expression for the variation in bath temperature as the regulator operates is not derived but experiment shows this variation to be small. The numerical calculations in the appendix indicate the advantages in regulation which may be gained by the use of an oscillating contact regulator. The principal advantages of this type of regulator over the fixed contact are:

1. A large range of available energy input and consequently a large range of regulation may be obtained without a sacrifice of closeness of regulation.
2. A bath temperature is obtained in which the periodic variations about the mean due to operation of the regulator are very greatly reduced.
3. The variation introduced by variations in the mercury surface due to soiling, sticking, etc., are largely eliminated.

It is clear that the advantages to be secured by the use of the oscillating contact type of regulator are due to the fact that the time at which a given movement of the meniscus may affect the energy input is rendered independent of the physical constants of the bath and dependent only upon the periodicity of the oscillating element of the regulator, and to the provision of a means for applying successive corrections at short time intervals to the value of the energy input instead of corrections at such longer time intervals as will permit of wider excursions of bath temperature above and below its mean value.

The conditions to be fulfilled for successful operation of an oscillating contact thermoregulator are:

1. The periodicity of the oscillation should be small in comparison

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[1] Equation numbers are taken from the order in which they appear in the appendix.

with the lag of the bath, say $^1/_5$ of this value, but not so small as to produce sustained waves on the mercury surface. There seems to be no advantage in reducing the period beyond that necessary to damp the periodic fluctions in bath temperature sufficiently to render their effects imperceptible.

2. The length of path of the oscillating element should be large in comparison with the movement required to make or break contact with the mercury surface; one mm. seems to be sufficient in most cases.

3. The energy distribution along the path of the oscillating element should be large in order that great range and close regulation may be secured under a wide range of external conditions. The upper limit to this energy distribution is fixed by the fact that the smallest amount of energy which may be supplied during a single cycle should not exceed that which is required during that cycle. Violation of this condition would result in a motion of the meniscus beyond the limits of the stroke of the oscillating element. An upper limit to the energy distribution lower than that indicated above may be imposed by the current carrying capacity of the relay contacts, etc.

It will be seen that the method of energy control exemplified in the oscillating contact thermoregulator can be applied readily to a wide variety of forms of manual or automatic control of various physical quantities.

Summary.

The paper describes a modification of the usual type of thermoregulator in which the fixed contact element is replaced by an oscillating contact element and shows that such a regulator will reduce the periodic variation of the bath temperature and the erratic variations due to variations of the mercury surface to a fraction of the values to be obtained with the usual form of thermoregulator. A periodic variation in bath temperature of less than 0.0001° as indicated by a temperature indicator having a natural period of about five seconds is easily obtained.

In addition, by use of an oscillating contact regulator, variations of mean bath temperature due to variations in external conditions are reduced below the values usually obtained.

Characteristic equations are derived for both the fixed and oscillating contact type of thermoregulator and these equations together with experimental evidence have been used to draw a comparison between the two types of regulator.

Appendix.

The effects of variations in external conditions upon the temperature of the bath will be considered, it being understood that in this discussion "temperature of bath" means specifically the temperature of that portion of the bath which is in immediate contact with the thermoregulator.

The relation between the temperature of this portion of the bath and the temperature of the bath as a whole is of course dependent upon the degree of mixing of the bath fluid, the relative positions of the heater and the thermoregulator and the degree of thermal contact between the bath, heater, thermoregulator and cooling surfaces. It is also assumed that the lag of the bath as a whole with respect to the heater is less than the lag of the thermoregulator with respect to the heater. If this were not the case, *i. e.*, if the thermoregulator were very closely coupled thermally with the heater, the effective heat capacity of the bath defining the operation of the thermoregulator would be less than the real heat capacity of the bath. Effects of exposed stem volume are neglected.

Fixed contact regulator:

Let K = cooling constant of bath, deg./sec. deg.

ϕ = the portion of the thermal head of the bath, *i. e.*, difference in temperature between the exposed portion of the bath and its surroundings, which is compensated by the thermoregulator. This does not include the portion of the thermal head which is compensated by the fixed heating. ϕ is considered positive when the bath loses heat to the surroundings.

$R_c = K\phi$ = cooling rate, deg./sec.

R_h = rate of temperature rise of bath when energy is being supplied, deg./sec.

Z = average temperature at which "make" and "break" occurs.

$\Delta\theta'$ = contact lag; the change in temperature required to change the contact of the thermoregulator from "make" to "break." This quantity has been found to vary rather erratically by as much as 50% under ordinary conditions.

$\Delta\theta_p$ = total change in temperature during one cycle of "make" and "break." Amplitude of the periodic oscillations of temperature.

θ = instantaneous temperature of bath.

θ_m = mean temperature of bath.

W = maximum electrical input supplied by regulator, watts; not average input.

M = heat capacity of bath, joules/deg.

t = time lag of thermoregulator, seconds. This quantity is defined as the number of seconds which would elapse between the time when the total energy input to the bath has reached a value corresponding to a given meniscus position and the time when the meniscus assumes this position, supposing the rate of energy input to be approximately constant.

Now, $R_c = K\phi$ and $R_h = W/M - R_c = W/M - K\phi$.

Then approximately

$$\theta_m = Z + t/2\,(R_h - R_c) = Z + t/2\,(W/M - 2K\phi).$$

Expressing finite changes, θ_m, in terms of changes in ϕ and W we have

$$\Delta\theta_m = \frac{t}{2M}\,\Delta W - tK\,\Delta\phi, \qquad (1)$$

which is the characteristic equation for the fixed contact thermoregulator.

The magnitude of the periodic change in bath temperature as the thermoregulator operates is

$$\Delta\theta_p = tR_c + tRh + \Delta\theta' = \frac{tW}{M} + \Delta\theta'. \qquad (2)$$

Oscillating contact regulator.

Let α = sensitivity of thermoregulator; the movement of meniscus in cm. per degree change in bath temperature.
θ = instantaneous bath temperature.
θ_m = mean bath temperature.
ϕ = thermal head as defined for fixed contact regulator.
y = length of path of oscillating contact in centimeters over which energy is being delivered to the bath.
W = electrical energy distribution along the path of the oscillating element expressed in watts/cm., *i. e.*, a change of 0.1 cm. in position of the meniscus would change the average power input by $W/10$ watts.

For equilibrium conditions it is evident that we must have the condition

$$y = \frac{MK\phi}{W}$$

Expressing finite changes in y in terms of finite changes in ϕ and W we obtain,

$$\Delta y = \frac{MK\,\Delta\phi}{W_2} - \frac{MK\phi_1\,\Delta W}{W_1W_2}, \qquad (3)$$

where the subscripts 1 and 2 denote values before and after the change considered has taken place:

Now by definition of α,

$$\Delta y = -\alpha\,\Delta\theta_m. \qquad (4)$$

Substituting this value for Δy in Equation 3 we have

$$\Delta\theta_m = \frac{MK\phi_1}{\alpha W_1 W_2}\,\Delta W - \frac{MK\,\Delta\phi}{\alpha W_2}, \qquad (5)$$

which is the characteristic expression for the oscillating contact thermoregulator.

As an experimental verification of the above equations observations were taken of the action of a thermoregulator operating first as a fixed and then as an oscillating contact type. The bath used was the water jacket of the calorimeter described in the Bureau of Standards, *Scientific Paper* 231.

The time lag of the thermoregulator with respect to the heater was obtained as follows: The energy input to the heater was changed from a value which gave a zero temperature rate to one which would give a fairly large rate and reckoning from the time at which this change was made data for the time-displacement curve of the mercury meniscus of the thermoregulator were taken. This curve was plotted and the constant displacement rate which was attained after about thirty seconds was projected back to cut the time axis. This intercept gave a single determination of the time lag of the bath. The final value was the mean of a number of such observations.

The other constants of the bath and regulator were determined in the usual manner and found to be as follows:

Heat capacity of bath, M, = 80,000 joules/deg.
Cooling constant of bath, K, = 0.000053 deg./sec./deg.
Time lag, t, = 8.4 seconds.
Sensitivity of regulator, α = 2.65 cm./deg.
Temperature change for "make" to "break" $\Delta\theta'$ = 0.0025° ± 0.0005.
Length of stroke of oscillating contact = 0.103 cm.
Period of oscillating contact = 1.7 seconds.

In order to secure changes in θ_m which could be observed with the desired accuracy it was necessary to change the working conditions of the thermoregulator very widely. It is to be understood that changes of this order of magnitude would seldom be met with in practice.

Observations were taken and calculations made as indicated in the tables below.

Fixed Contact Regulator.

$$\Delta\theta_m = t/2M\ \Delta W - tK\,\Delta\phi. \qquad \Delta\theta_p = t\ W/M + \Delta\theta'.$$

No.	W watts.	ΔW watts.	ϕ. Deg.	$\Delta\varphi$. Deg.	$\Delta\theta_p$ obs. Deg.	$\Delta\theta_p$ calc. Deg.	$\Delta\theta_m$ obs. Deg.	$\Delta\theta_m$ calc. Deg.
1...........	50	+ 50	—	0	0.0070	0.0078	+0.0027	0.0026
2...........	100	+ 50	—	0	0.0120	0.0130	+0.0025	+0.0026
3...........	150	—100	—	0	0.0170	0.0180	—0.0050	—0.0052
4...........	50	— 20	—	0	0.0073	0.0078	—0.0010	—0.0011
5...........	120	— 40	19	—10.5	0.0150	0.0150	—0.00021	—0.0023
6...........	70	+ 50	16	+3.0	0.0085	0.0076	+0.0033	+0.0029
7...........	125	— 37	19	—9.8	0.0170	0.0160	—0.0020	—0.0021

Oscillating Contact Regulator.

$$\Delta\theta_m = \frac{MK\phi_1}{\alpha W_1 W_2}\,\Delta W - \frac{MK}{\alpha W_2}\,\Delta\phi.$$

No.	W_1 watts/cm.	W_2 watts/cm.	ΔW watts/cm.	ϕ_1. Deg.	$\Delta\phi$. Deg.	$\Delta\theta_m$ obs. Deg.	$\Delta\theta_m$ calc. Deg.
1.........	680	1160	480	10.0	0	+0.0096	0.0098
2.........	775	1450	675	10.0	0	+0.0095	0.0096
3.........	970	1160	190	18.5	0	+0.0050	0.0050
4.........	1160	1160	0	10.0	—2.4	0.0037	0.0033
5.........	675	675	0	10.0	—2.0	0.0043	0.0048
6.........	970	970	0	18.5	—10.0	0.0170	0.0165
7.........	1160	1160	0	18.5	—7.7	0.0110	0.0106

The agreement between observed and calculated values seems good in view of the magnitudes and nature of the quantities observed.

The apparatus described above has constants which are more favorable to the fixed than to the oscillating contact type of thermoregulator, furthermore, it is hardly typical in that the heat capacity is rather larger and the time lag smaller than is usual. Constants which are more nearly representative of the average case will be assumed and the variation in θ_m with varying working conditions will be calculated.

M = 20,000 joules/deg.
ϕ = 5° C.
α = 20 cm./deg. (270 cc. bulb, 1 mm. capillary).
K = 0.0001 deg./sec./deg.
t = 15 seconds.
W = 1000 watts/cm. for oscillating contact.
W = 10 watts for fixed contact.
$\Delta\theta'$ = 0.0004 deg.

Assuming a change of 1° in ϕ. For fixed contact, from Equation 1

$$\Delta\theta_m = 15 \times 0.0001 \times 1 = 0.0015^\circ.$$

For oscillating contact, from Equation 5

$$\Delta\theta_m = \frac{20{,}000 \times 0.0001 \times 1}{20 \times 1{,}000} = 0.0001^\circ.$$

The amplitude of the periodic oscillation of temperature is for the fixed contact thermoregulator, from Equation 2.

$$\Delta\theta_p = \frac{15 \times 8}{20{,}000} + 0.004 = 0.0064^\circ.$$

Experiment has shown that the amplitude of the periodic oscillation of temperature produced by the oscillating contact regulator appears less than 0.0001°.

Assuming the energy input to change by 1% of itself; a change which might be due to a change in the voltage of the supply circuit.

For fixed contact, from Equation 1

$$\Delta\theta_m = \frac{15}{2 \times 20{,}000}(10 \times 0.01) = 0.000038^\circ.$$

For oscillating contact, from Equation 5

$$\Delta\theta_m = \frac{20{,}000 \times 0.0001 \times 5}{20 \times 1000 \times 990} \times (1000 \times 0.01) = 0.000005^\circ.$$

Hence the effect of small changes in power input would be negligible in either case.

In conclusion, the author wishes to express his appreciation of the very great assistance rendered by Mr. M. S. Van Dusen of the Bureau of Standards in the preparation and verification of this discussion.

WASHINGTON, D. C.